Numerical problems based on Isothermal and adiabatic process

B.Sc . Part- I. (Physics honours& Subsidiary)

By

Dr.Rakesh Kumar Ranjan

Assistant Professor

Department of Physics A.S.COLLEGE Bikramganj, V.K.S. University,Ara

Email id:-ranjan.rakesh130@gmail.com

Mob : 870 930 4765, 9534531001

Numerical problems based on Isothermal and adiabatic process.

Ex. 1:- **A certain gas at atmospheric pressure is compressed adiabatically so that its volume becomes half of its original volume. Calculate the resulting pressure in Nm^{-2}.**

Take □ = 1.4, for air,

***Sol^n*:-** **Let the original Volume.**

$$V_1 = V$$

□ Final Volume, $V_2 = V/2$

Initial pressure P_1 = 0.76 meter of Hg column

Let P_2 be the final pressure after compression.

As the change is adiabatic,

$$\therefore \quad P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

or

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = P_1\left(\frac{V}{V/2}\right)^{1.4}$$

$$P_2 = 0.76\times(2)^{1.4}$$

$$P_2 = 2.00 \text{ metre of Hg Column}$$

As, $P = h \rho g$

$\therefore$ $P_2 = 2.00 \times (13.6 \times 10^3) \times 9.8\ Nm^{-2}$

$P_2 = 2.672 \times 10^5\ Nm^{-2}$

Ex. 2:-

A gram molecule of a gas at 127^0C expands isothermally until its volume is doubled. Find the amount of work done and heat absorbed.

Sol^n:-

Here, temperature of the gas,

$T = 273 + 127 = 400\ K$

Let initial volume of the gas, $V_1 = V$

$\therefore$ **Final volume of the gas,** $V_2 = 2V$

In an isothermal expansion,

$$\text{Work done } (W) = 2.3026 \ RT \log_{10} \frac{V_2}{V}$$

$$= 2.3026 \times 8.3 \times 400 \times \log_{10} \frac{2V}{V}$$

$$= 2.3026 \times 8.3 \times 400 \times 0.3010$$

Or

$$W = 2.30 \times 10^3 \text{ joule}$$

If H is the amount of heat absorbed,

$$H = \frac{W}{J} = \frac{2.30 \times 10^3}{4.2} = 548\, cal.$$

Ex. 3:- **A cylinder containing one gram molecule of the gas was compressed adiabatically until its temperature rose from 27^0C to 97^0C. Calculate the work done and heat produced in the gas (□ = 1.5).**

***Sol^n*:-** **Here, initial temperature,**
$T_1 = 27^0C = 273 + 27 = 300$ K

final temperature, T2 = 97^0C = 273 + 97 = 370 K When a

gas is compressed adiabatically, work done on

the gas is given by

$$W = \frac{R}{(1-\gamma)}(T_2 - T_1)$$

$$= \frac{8.3 \times (370-300)}{1-1.5}$$

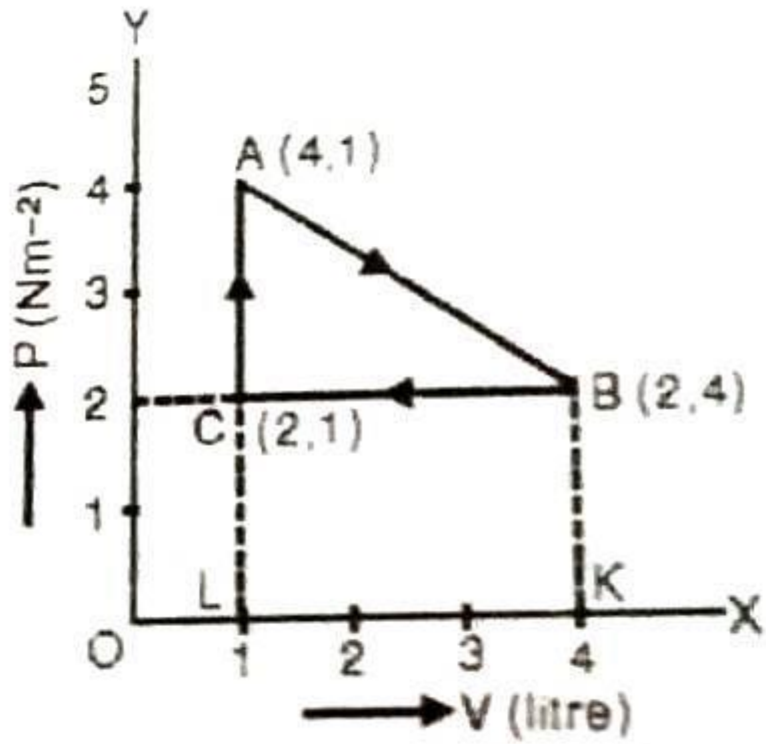
or $$W = -11.62 \times 10^2 \, J$$

 Heat produced,

$$H = \frac{W}{J} = \frac{11.62 \times 10^2}{4.2} = 276.7 \, cal.$$

Ex. 4:- **The *P-V* diagram for a cyclic process is a triangle *ABC* drawn in order. The co-ordinates of *A, B C* are (4,1), (2,4) and (2,1). The co-ordinates are in the order (P,V). Pressure is in Nm^{-2} and volume is in liter. Calculate work done during the process from *A* to *B, B* to *C* and *C* to *A*. Also calculate work done in the complete cycle.**

***Sol^n*:-** **The *P-V* diagram drawn as per the questions is shown in Fig.**



(i) **Work done during the process from *A* to *B* (expansion)**

W_{AB} = + area ABKLA

= area of □ABC + area of rectangle BCLK

$$W_{AB} = \frac{1}{2} BC \times AC + KL \times LC$$

$$Now, \quad BC = KL = 4 - 1 = 3\,litre$$

$$= 3 \times 10^{-3}\, m^3$$

AC = 4 - 2 = 2 *Nm^{-2}*

LC = 2 - 0 = 2 *Nm^{-2}*

$$\therefore \quad W_{AB} = \frac{1}{2} \times 3 \times 10^{-3} \times 2 + 3 \times 10^{-3} \times 2$$

$$W_{AB} = 9 \times 10^{-3} J$$

(ii) **Work done during the process from *B* to *C* (compression) is**

$$W_{BC} = - \text{area } BCLK = - KL \times LC$$

$$= -3 \times 10^{-3} \times 2 = -6 \times 10^{-3} \text{ J.}$$

(iii) **Work done during the process from *C* to *A* As there is no change in volume of the gas in this process, therefore, $W_{CA} = 0$**

Net work done in the complete cycle

$$W = W_{AB} + W_{BC} + W_{CA}$$

$$= 9 \times 10^{-3} + (-6 \times 10^{-3}) + 0$$

$$= 3 \times 10^{-3} \text{ J.}$$

Note :- **The same value of W is obtained when we take area of the triangle *ABC* repressing the cyclic process.**

Thank You

Dr.Rakesh Kumar Ranjan