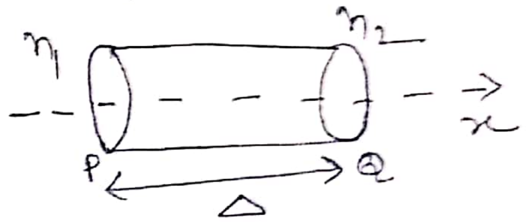
Einstein's Theory of Brownian motion

According to Einsten's theory of Brownian motion, the particle tend to diffuse into the medium in course of time. Consequently, the diffusion coefficient must be related to the Brownian movement.

The diffusion coefficient can be calculated in two different ways:

(i) From the irregular motion of the suspended particles.

(ii) From the difference in osmotic pressure caused by the differences in concentration of the suspended particles



$n_1, n_2 \rightarrow$ molecular concentration at face P & Q

The number of particles crossing the surface P to the right in time, τ $= \frac{1}{2} n_1 A \Delta$

similarly for Q surface, $= \frac{1}{2} n_2 A \Delta$

The excess number of particles $= \frac{1}{2}(n_1 - n_2) A \Delta$

from the defination of diffusion coefficient

$$\frac{1}{2}(n_1 - n_2) A \Delta = -D \frac{dn}{dx} \tau A \quad \text{--- (1)}$$

$$(n_1 - n_2) = -\Delta \frac{dn}{dx} \quad \text{--- (2)}$$

$$-\frac{1}{2}\Delta^2 \left(\frac{dn}{dx}\right) = -D\left(\frac{dn}{dx}\right)\tau$$

$$\Delta^2 = 2D\tau \Rightarrow D = \frac{\Delta^2}{2\tau} \quad \text{--- (3)}$$

If P_1 & P_2 are the osmotic pressure at the end P and Q, then

$$P_1 = n_1 KT$$
$$P_2 = n_2 KT$$

Thus the cylinder experiences a resultant force,

$$(P_1 - P_2)A = (n_1 - n_2) KTA$$

The number of particles in the cylinder

$$= nA\Delta$$

The force acting on a single particle

$$f' = \frac{(n_1 - n_2) KTA}{nA\Delta}$$

$$f' = -\left(\frac{dn}{dx}\right)\left(\frac{KT}{n}\right) \quad \text{(from eq (3))} \quad -④$$

$$f' = 6\pi\eta a v \quad -⑤$$

equation eq(4) & (5), equating;

$$nv = -\left(\frac{KT}{6\pi\eta a}\right)\left(\frac{dn}{dx}\right) \quad -⑥$$

&

$$nv = -D\left(\frac{dn}{dx}\right) \quad -⑦$$

Equating (6) & (7), we get

$$-D\left(\frac{dn}{dx}\right) = -\left(\frac{KT}{6\pi\eta a}\right)\left(\frac{dn}{dx}\right)$$

$$D = \frac{KT}{6\pi\eta a} \quad \& \quad D = \frac{\Delta^2}{2\tau} \quad ⑧$$

$$\Delta^2 = \frac{RT}{N}\left(\frac{1}{3\pi\eta a}\right)\tau \qquad [R = KN]$$