Brownian Motion

The spontaneous and continous movement of the particles in zig-zag way is called as Brownian motion. The motion of the particles becomes more violent with increase in temperature. The phenomenon of Brownian motion gives a clear picture of the gaseous state of matter. The laws of kinetic theory of gases are applicable to Brownian particle also.

Salient features of Brownian Motion :-

(i) The motion of each particle is completely irregular

(ii) The motion is continous and random.

(iii) The smaller particles appear to more agitated than larger ones.

(iv) The motion is independent of the nature of the suspended particles.

(v) The motion becomes more violent on increasing temp.

(vi) The laws of kinetic theory of gases are applicable to Brownian motion too.

Langevin's theory of Brownian Motion

According to langevin theory, the force experienced by a suspended particle is of two types:

1) Frictional force ; $f\left(\frac{dx}{dt}\right) = 6\pi\eta a v$

2) Force due to all external influences of the surrounding fluid.

$$m\left(\frac{d^2x}{dt^2}\right) = -f\left(\frac{dx}{dt}\right) + F \quad \text{—} (1)$$

↑ frictional force; F ← Combined force due to various effects

multiply eq (1) by x, we get

$$mx\left(\frac{d^2x}{dt^2}\right) = -f(x)\left(\frac{dx}{dt}\right) + Fx \quad \text{—} (2)$$

Also

$$\frac{d}{dt}(x^2) = 2x\left(\frac{dx}{dt}\right) \text{ and } x\frac{dx}{dt} = \frac{1}{2}\frac{d}{dt}(x^2)$$

$$\frac{d}{dt}\left[\frac{d}{dt}(x^2)\right] = 2\left[x\left(\frac{d^2x}{dt^2}\right) + \left(\frac{dx}{dt}\right)^2\right]$$

$$x\frac{d^2x}{dt^2} = \frac{1}{2}\frac{d}{dt}\left[\frac{d}{dt}(x^2)\right] - \left(\frac{dx}{dt}\right)^2 \quad \text{—} (3)$$

Putting these value in eq (2), we get

$$\frac{m}{2}\frac{d}{dt}\left[\frac{d}{dt}(x^2)\right] - m\left(\frac{dx}{dt}\right)^2 = -\frac{f}{2}\frac{d}{dt}(x^2) + Fx \quad \text{—} (4)$$

Now for all particle, the mean value can be written as

$$\frac{m}{2}\frac{d}{dt}\left[\frac{d}{dt}(x^2)\right] - m\overline{\left(\frac{dx}{dt}\right)^2} = \frac{f}{2}\overline{\frac{d}{dt}(x^2)} + \overline{Fx} \quad \text{—} (5)$$

If we assume, $\overline{Fx} = 0$ & $m\left(\frac{dx}{dt}\right)^2 = KT$
and $\frac{d}{dt}(x^2) = \frac{d}{dt}(\bar{x^2}) = U$

$$\frac{m}{2}\left(\frac{du}{dt}\right) + \frac{f(u)}{2} = KT$$

$$\frac{du}{dt} + \left(\frac{f}{m}\right)U = \frac{2KT}{m}$$

The solution of above equation is given by

$$U = \frac{2KT}{f} + A\left[e^{-f/m\,t}\right]$$

If $f/m \to \infty$ then; $U = \frac{2KT}{f} = \frac{d\overline{x^2}}{dt}$

from $t = 0$ to $t = \tau$

$$\overline{x^2} - \overline{x_0^2} = \left(\frac{2KT}{f}\right)\tau$$

and at $t = 0$ & $x_0 = 0$

$$\overline{\Delta x^2} = \left(\frac{2KT}{f}\right)\tau \qquad (6)$$

And.

$$f\left(\frac{dx}{dt}\right) = 6\pi\eta a v = 6\pi\eta a\left(\frac{dx}{dt}\right)$$

$$f = 6\pi\eta a$$

$$\overline{\Delta x^2} = \frac{2KT\tau}{6\pi\eta a} = \frac{KT\tau}{3\pi\eta a}$$

$$\left[\overline{\Delta x^2}\right]^{1/2} = \frac{\sqrt{KT}\,(\tau)^{1/2}}{(3\pi\eta a)^{1/2}} \qquad (7)$$

from this equation,

$$(\overline{\Delta x^2})^{1/2} \propto \tau^{1/2} \qquad (8)$$

$$\propto \frac{1}{\eta^{1/2}} \qquad (9)$$

Thus effect of temperature is not very large because from eq (8) however viscocity decrease with increase in temperature eq (9).

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