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lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$$

There are n such equations in all. when the forces are derivable from a scalar potential function V,

$$\vec{F}_i = -\nabla_i V$$

Then the generalized forces can be written as

$$Q_j = \sum_i \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j} = -\sum_i \nabla_i V \cdot \frac{\partial \vec{r}_i}{\partial q_j}$$

which is exactly the same expression for the partial derivative of a function $-V(\vec{r}_1, \vec{r}_2, --\vec{r}_N, t)$ with respect to q_j

$$Q_j = -\frac{\partial V}{\partial q_j}$$

The equation can be rewritten as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial (T-V)}{\partial q_j} = 0$$

The equations of motion in the form are not necessarily restricted to conservative systems;

$$\frac{d}{dt}\left(\frac{\partial (T-V)}{\partial \dot{q}_j}\right) - \frac{\partial (T-V)}{\partial q_j} = 0$$

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Defining a new function, the lagrangian

$$L = T - V$$

The eqs: become

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$$

This is known as lagrange's equations
The particular set of equations of motion leads to the given generalised co-ordinates.
If $L(q,\dot{q},t)$ is an approximate lagrangian and $F(q,t)$ is any differentiable function of the generalised coordinates and time, then

$$L'(q,\dot{q},t) = L(q,\dot{q},t) + \frac{dF}{dt}$$

It is also same equation of motion.