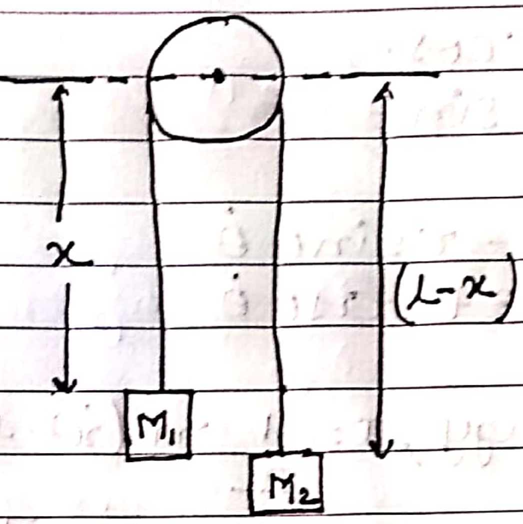
Date: __/__/____

Atwood's machine



There is only one independent coordinate x, the position of the other weight being determined by the constraint that the length of the rope between them is l.

The Potential energy is

$$V = -M_1 g x - M_2 g(l-x)$$

while the kinetic energy is

$$T = \frac{1}{2}(M_1 + M_2)\dot{x}^2$$

$$L = T - V$$

$$L = \frac{1}{2}(M_1 + M_2)\dot{x}^2 + M_1 g x + M_2 g(l-x)$$

equation of motion :-

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

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$$\frac{\partial L}{\partial \dot{x}} = \frac{1}{2} \times 2 (M_1 + M_2)\, \dot{x} = (M_1 + M_2)\, \dot{x}$$

$$\frac{\partial L}{\partial x} = M_1 g + M_2 g (-1) = M_1 g - M_2 g$$

so that, we have

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$(M_1 + M_2)\, \ddot{x} - (M_1 - M_2) g = 0$$

$$\ddot{x} = \frac{(M_1 - M_2)\, g}{(M_1 + M_2)}$$