

Velocity - Dependent Potentials and the Dissipation function

Lagrange's equation can be put in the form even if there is no potential function, V . The generalized forces are obtained from a function $U(q_j, \dot{q}_j)$

$$Q_j = - \frac{\partial U}{\partial q_j} + \frac{d}{dt} \left(\frac{\partial U}{\partial \dot{q}_j} \right)$$

If the Lagrangian is given by:-

$$L = T - U$$

U may be called a "generalized potential" or "velocity - dependent potential".

Simple Applications of the Lagrangian formulation

1. Motion of one particle with cartesian coordinates:-

The kinetic energy, $T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$

$$U = 0$$

$$L = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

The equation of motion :-

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = 0$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} \quad , \quad \frac{\partial L}{\partial \dot{y}} = m\dot{y} \quad , \quad \frac{\partial L}{\partial \dot{z}} = m\dot{z}$$

$$\frac{d}{dt} (m\dot{x}) = F_x \quad , \quad \frac{d}{dt} (m\dot{y}) = F_y \quad , \quad \frac{d}{dt} (m\dot{z}) = F_z$$