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Langrangian of particle by using Plane Polar coordinates

The kinetic energy, $T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial y} = \frac{\partial T}{\partial z} = 0$$

$$\frac{\partial T}{\partial \dot{x}} = m\dot{x} \;,\; \frac{\partial T}{\partial \dot{y}} = m\dot{y} \;,\; \frac{\partial T}{\partial \dot{z}} = m\dot{z}$$

and the equations of motion are

$$\frac{d}{dt}(m\dot{x}) = F_x \;,\; \frac{d}{dt}(m\dot{y}) = F_y \;,\; \frac{d}{dt}(m\dot{z}) = F_z$$

The langrangian equation of motion :-

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$L = T - V \quad , \quad V = 0$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = 0$$

$$m\ddot{x} = 0$$

$$\ddot{x} = 0$$

Similarly

$$\ddot{y} = 0$$

$$\ddot{z} = 0$$

In Plane Polar coordinates

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\dot{x} = \dot{r}\cos\theta - r\sin\theta\,\dot{\theta}$$
$$\dot{y} = \dot{r}\sin\theta + r\cos\theta\,\dot{\theta}$$

Kinetic energy, $T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2)$

$$T = \frac{1}{2} m \left[\dot{r}^2 + (r\dot{\theta})^2\right]$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{r}}\right) = \frac{1}{2} \times 2\, m\ddot{r} = m\ddot{r}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = \frac{1}{2} m \times 2\, r\ddot{\theta} = m r\ddot{\theta}$$

$$\frac{\partial L}{\partial r} = \frac{1}{2} m \times 2\, (r\dot{\theta}) = m\dot{\theta}$$

$$\frac{\partial L}{\partial \theta} = 0$$

$$m\ddot{r} - m\dot{\theta} = 0$$
$$\ddot{r} = \dot{\theta} \quad \text{—(1)}$$

$$m r\ddot{\theta} = 0$$
$$\ddot{\theta} = 0 \quad \text{—(2)}$$