$$\ddot{x} = \frac{(M_1 - M_2)}{(M_1 + M_2)} g$$

3. A bead (or ring) sliding on a uniformly rotating wire in a force-free space.
The wire is straight, and is rotated uniformly about some fixed axis perpendicular to the wire.
The transformation equation explicity contain the time.

$$x = r\cos\omega t$$
$$y = r\sin\omega t$$

$$\dot{x} = \dot{r}\cos\omega t - r\sin\omega t(\dot{\omega}) \quad \Big| \quad \dot{y} = \dot{r}\sin\omega t + r\cos\omega t\,\dot{\omega}$$

It is simple to take over.

$$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\omega}^2)$$

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Note that T is not a homogeneous quadratic function of the generalized velocities since there is now a term involving $\dot{r}$. The equation of motion is then

$$m\ddot{r} - mr\omega^2 = 0$$

or

$$\ddot{r} = r\omega^2$$

It is the familiar simple harmonic oscillator equation with a change of sign.
The solution $r = e^{\omega t}$ for a bead initially at rest on the wire shows that the bead moves exponentially outwards.
Again the method cannot finish the force of constraints that keeps the bead on the wire. The angular momentum $L = mr^2\omega = m\omega r_0^2 e^{2\omega t}$
Provides the force, $F = N/r$ which produces the constraint force,

$$F = 2mr_0\omega^2 e^{\omega t}$$ acting Perpendicular to the wire and the axis of rotation.